

PlumeSim

*A simulation approach to
autonomous adaptive plume tracking
with multiple AUVs*

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Goals

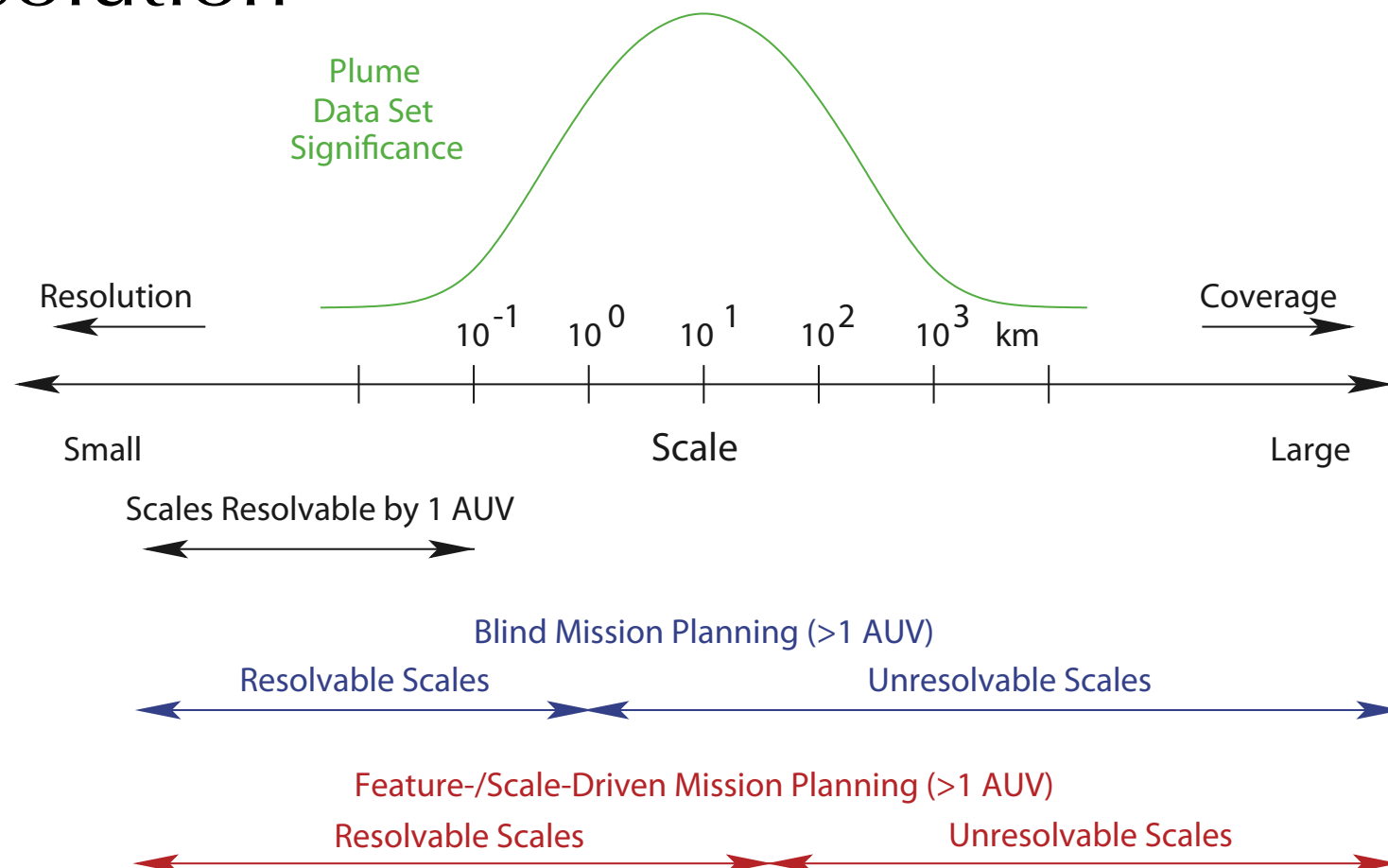
- Simulate a plume's shape as a sum of Fourier orders
- Sample the plume boundary with multiple AUVs
- Reconstruct the plume shape from AUV sample points
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- Analyze errors between simulated plume and AUV-estimated plume

- Predict & track hazardous underwater plumes: oil spills, harmful algal blooms, etc.



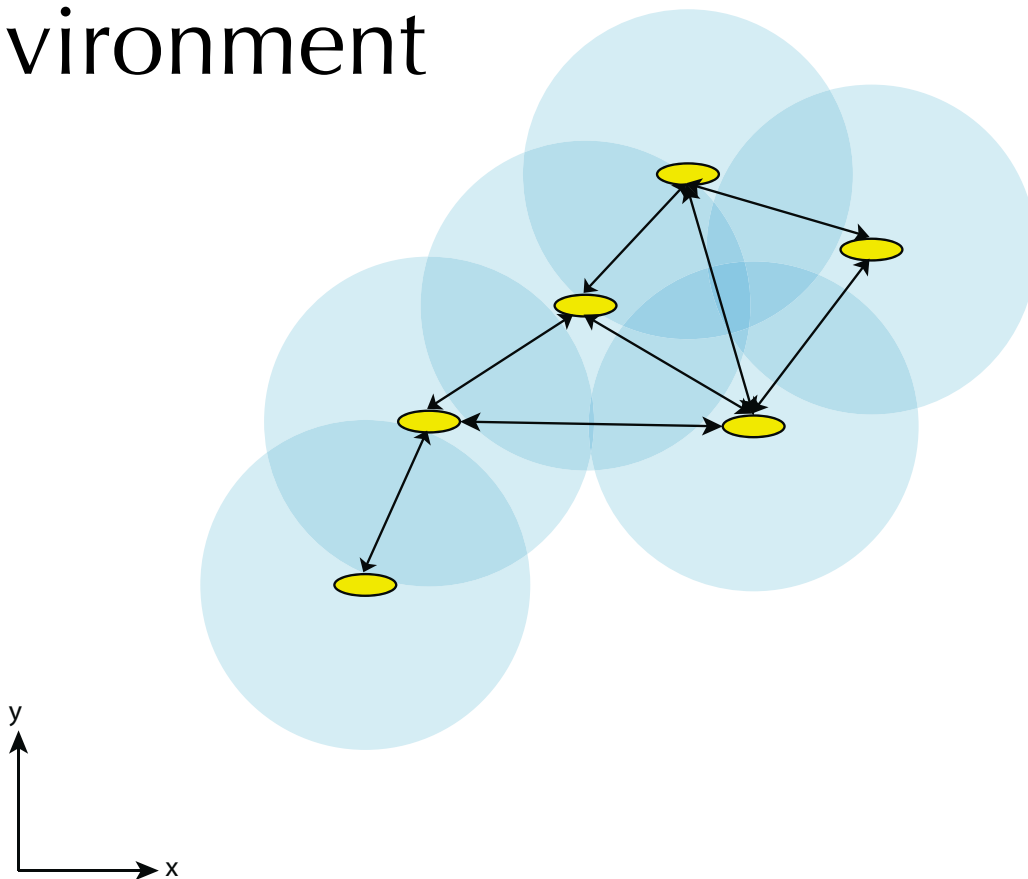
Motivation

- Spatiotemporal aliasing problem (large plumes, small AUVs), coverage vs. resolution



Motivation

- Make use of recent developments in AUV communications & autonomy to enable multi-AUV sensing of the ocean environment



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'Actual' Plume Construction: Sum of Fourier Orders

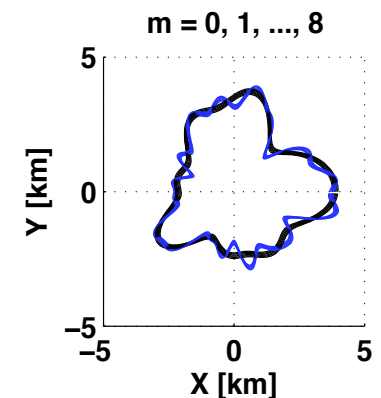
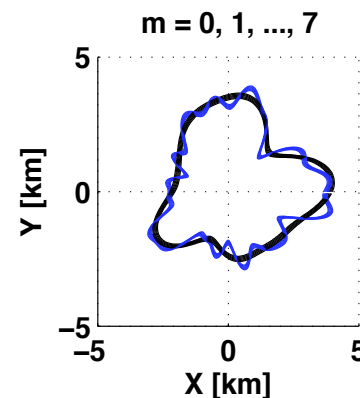
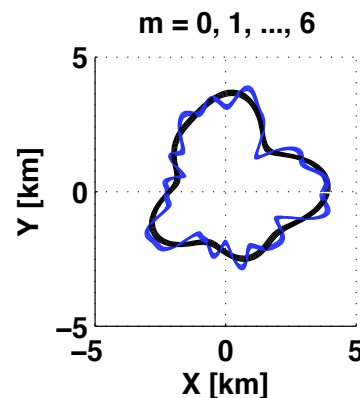
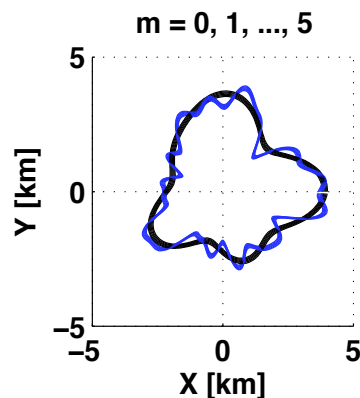
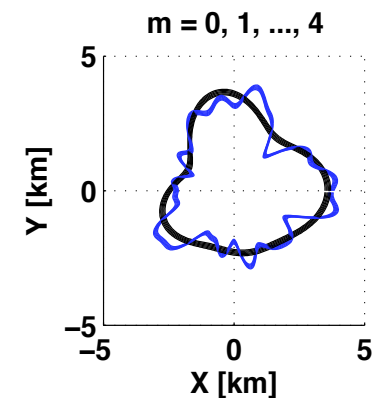
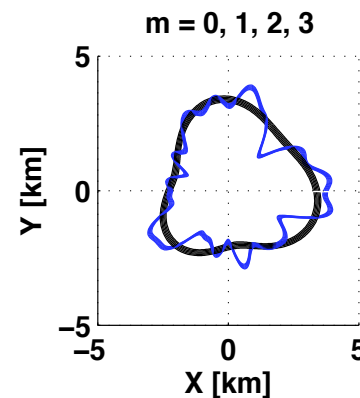
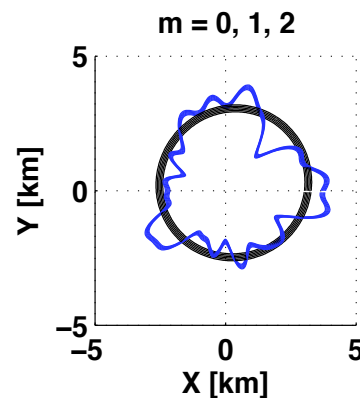
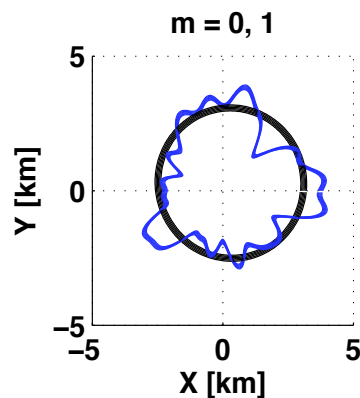
$$\Phi_M = \sum_{m=0}^M [A_m * \cos(m\theta + \phi_m)] + R, \quad (1)$$

- Construction in polar coordinates
- M = max # of Fourier orders
- $\Phi_M(\Theta)$ = radial distance of plume boundary from the origin at azimuth angle Θ
- Θ = azimuth angle about the plume origin
- R = radius of plume before perturbation
- A_m = amplitude perturbation for order m (random, bounded to 20% of R)
- ϕ_m = phase perturbation for order m (random, on $[0, 2\pi)$)

Simulated Plumes

‘Actual’ plume construction:

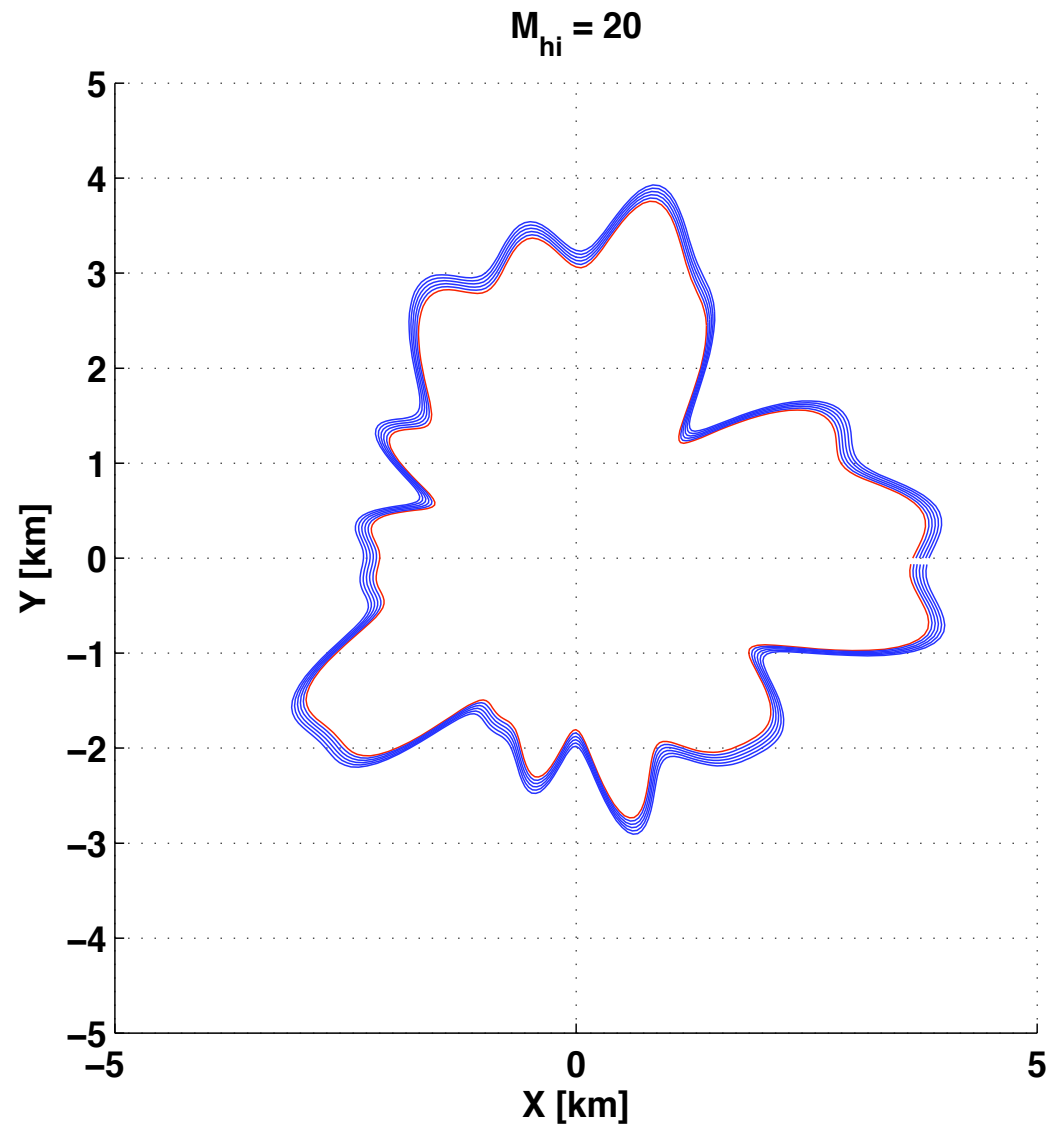
- $R = 5$ km
- $M = 1, 2, \dots, 8, \dots$ (black)



Simulated Plumes

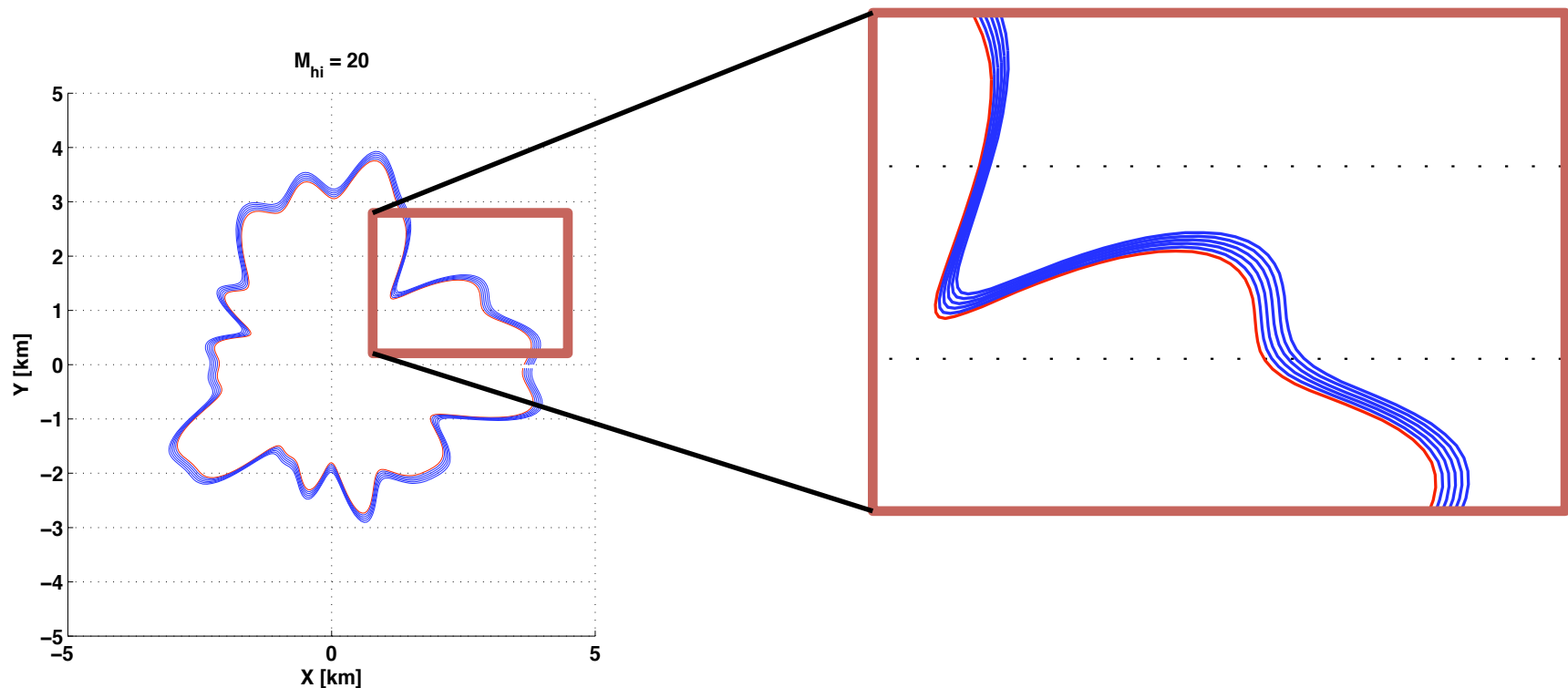
'Actual' plume:

- $R = 5$ km
- Max # of Fourier modes, $M_{hi} = 20$



Time Variation

- Increase the amplitude of each Fourier order linearly in time to simulate plume spreading (from diffusion/turbulence)



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Introduce AUVs

- AUVs sample points on the original plume boundary
- Evenly space the AUVs about the plume origin, on the boundary, with navigation error of
 - $\pm 10^\circ$ in azimuthal position
 - ± 500 m in radial position

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Estimating the Plume from AUV Samples

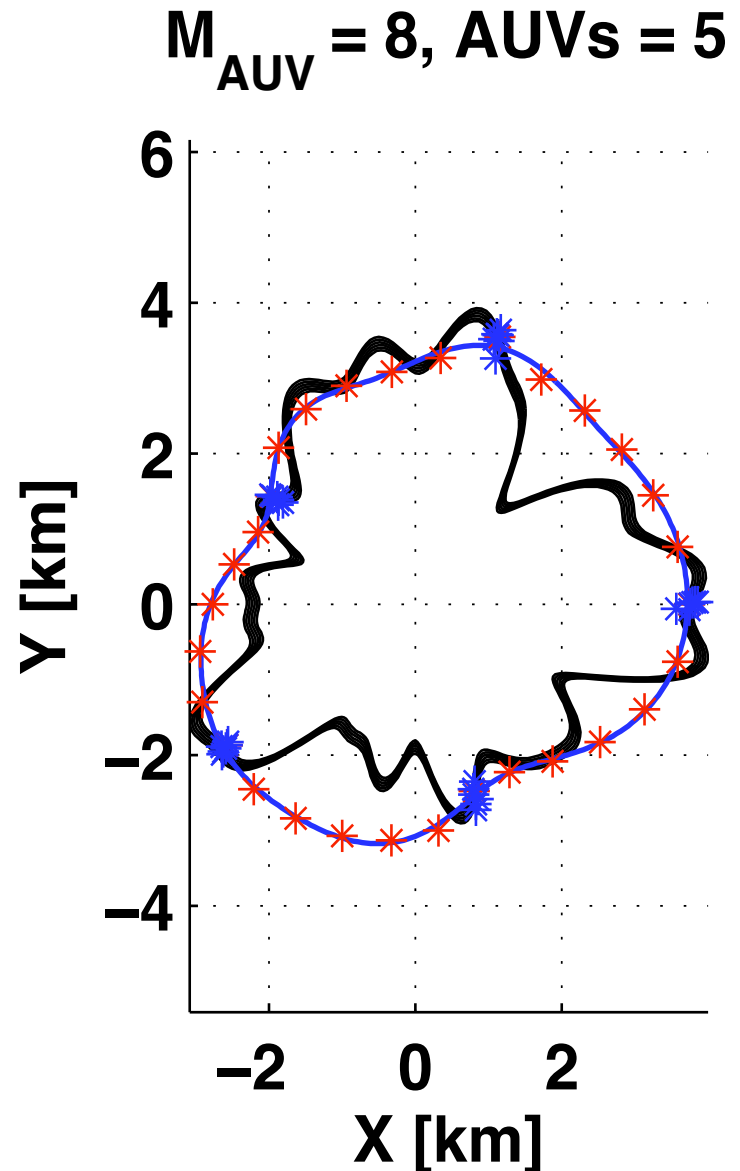
- Say, AUVs sample the moving plume edge every 2 minutes, and
- the plume is moving slowly enough that all samples in the last 10 minutes will give a decent estimate of the average plume boundary.

Estimating the Plume from AUV Samples

[blue stars]

- If we have 5 AUVs in the water,
- this gives us 30 samples total (at 0, 2, 4, ..., 10 min).
- Thus, we can solve for up to 14 Fourier orders!

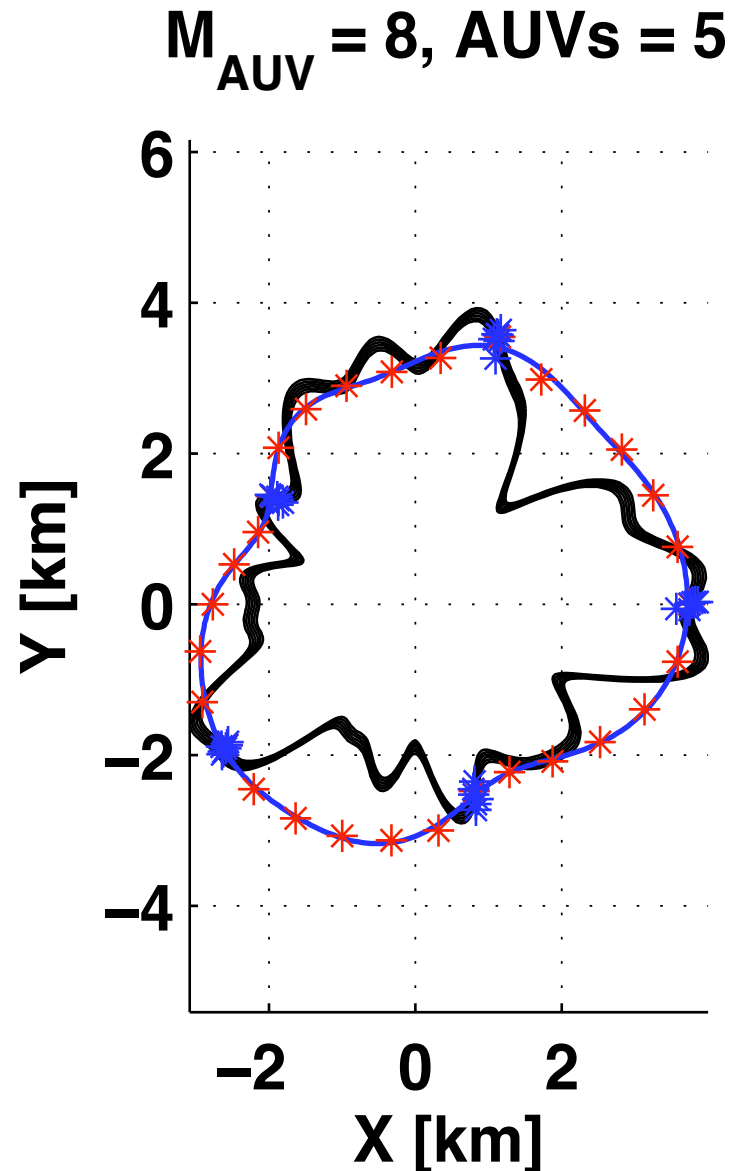
$$M \leq \text{floor}\{(N_{\text{samples}} - 1)/2\}$$



Estimating the Plume from AUV Samples

[red stars]

- Interpolate these samples to 30 evenly-spaced points about the plume (I use a cubic interpolation, but there are probably more robust methods out there).



Estimating the Plume from AUV Samples

- Back out estimates of the ‘actual’ plume coefficients (A_m & ϕ_m) using Fourier analysis on the ($N_{AUV} = 30$) interpolated AUV positions:

$$\Phi_{avg} \approx R \quad (2)$$

$$A_{AUV,m=0} = \frac{1}{2} * \frac{|fft(\Phi_{AUV,m=0}|\theta_{AUV})|}{N_{AUV}/2} - \Phi_{avg} \quad (3)$$

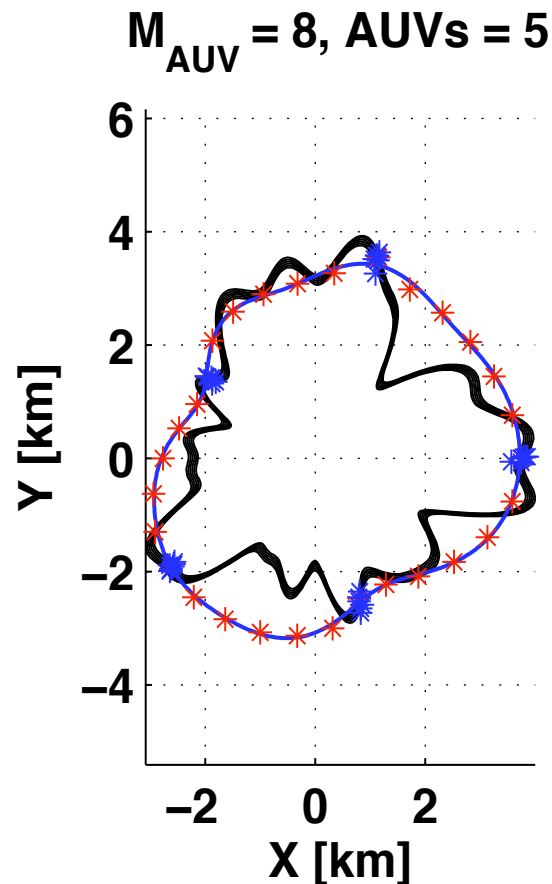
$$A_{AUV,m=1:M} = \frac{|fft(\Phi_{AUV,m=1:M}|\theta_{AUV})|}{N_{AUV}/2} \quad (4)$$

$$\phi_{AUV,m=0:M} = angle[fft(\Phi_{AUV,m=0:M}|\theta_{AUV})] \quad (5)$$

Estimating the Plume from AUV Samples

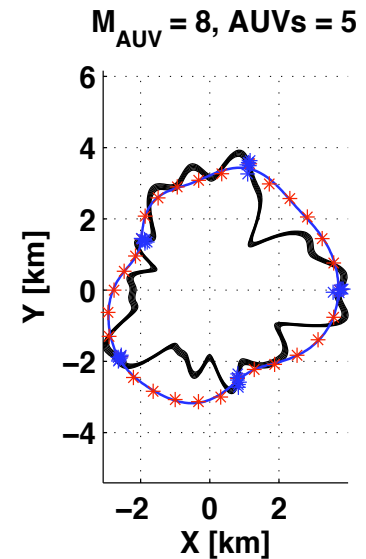
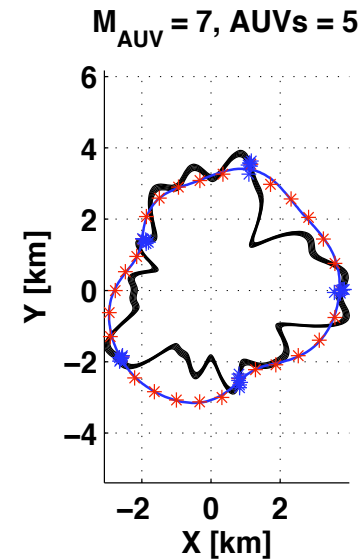
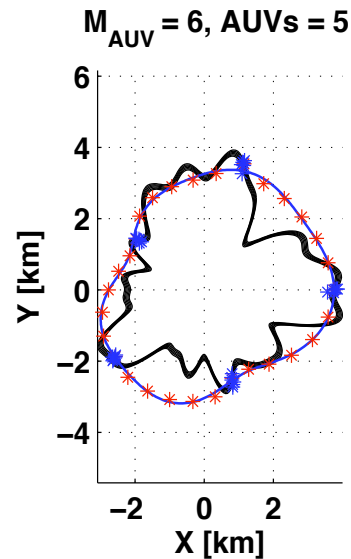
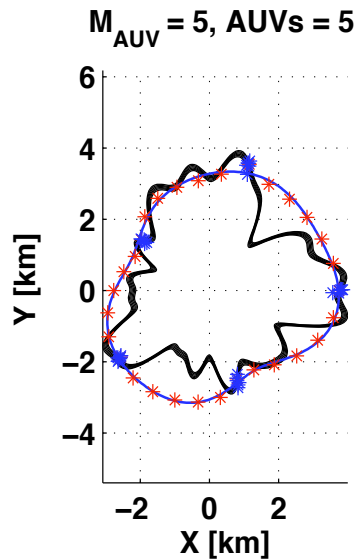
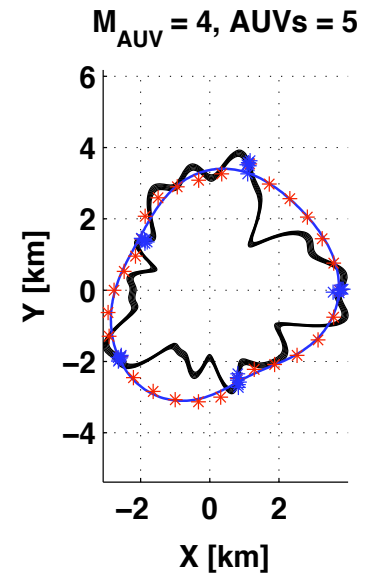
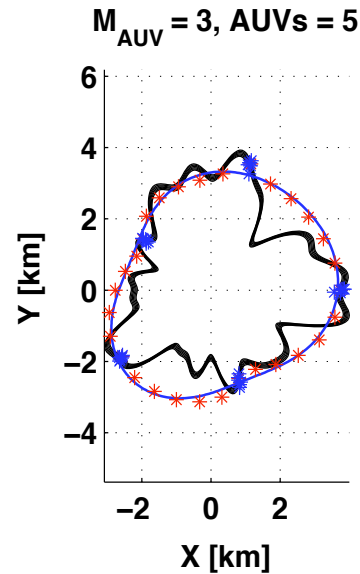
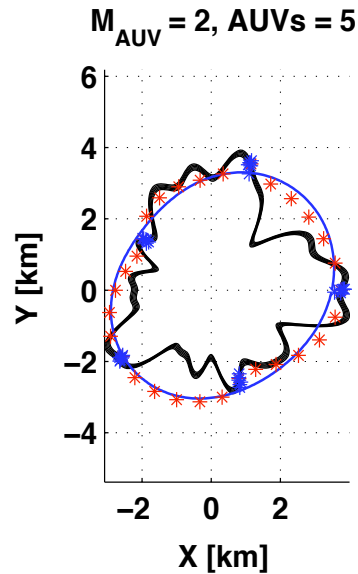
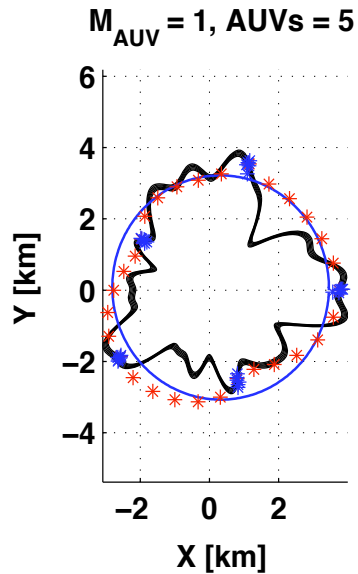
- Reconstruct the plume [blue line]:

$$\Phi_{AUV,M} = \sum_{m=0}^M [A_{AUV,m} * \cos(m\theta_{AUV} + \phi_{AUV,m})] + \Phi_{avg}. \quad (6)$$



Estimating the Plume from AUV Samples

- $M_{\text{AUV}} = 1, 2, \dots, 8$ [blue line]



Goals

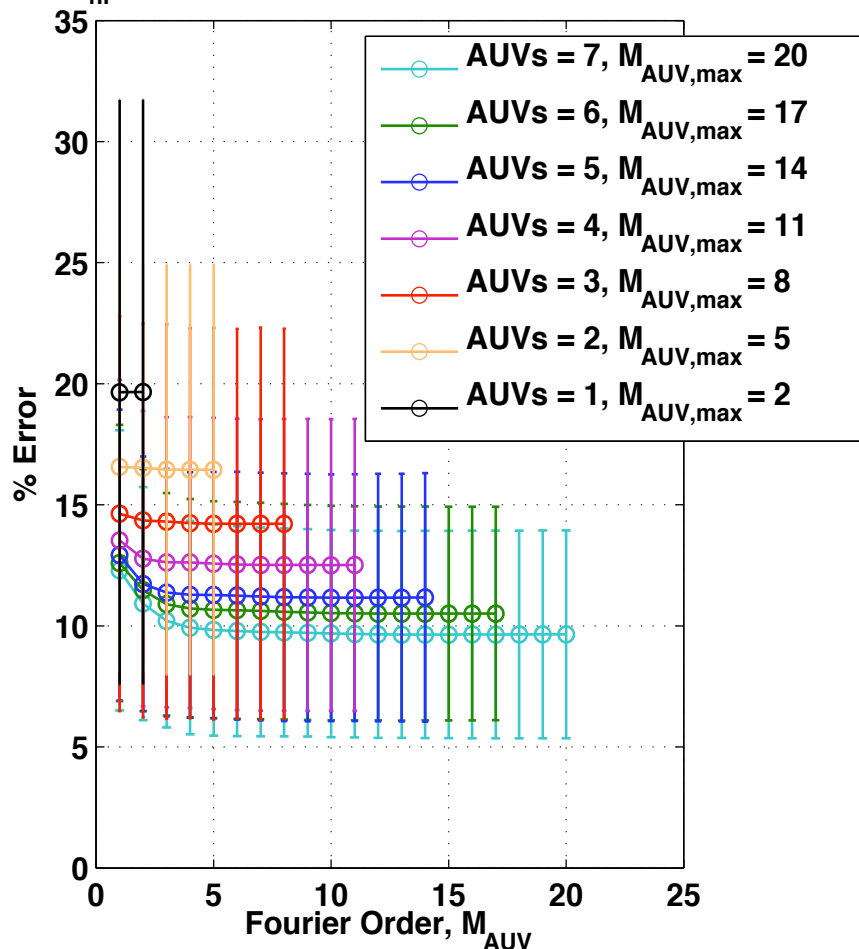
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Evaluate Errors

$$\%Error_{plume} = \frac{|\Phi_{estimated} - \Phi_{actual,time-avg}|}{\Phi_{actual,time-avg}}. \quad (7)$$

Mean and Standard Deviation of % Error (Estimated vs. Actual Plume)
for Each Fourier Order

$M_{hi} = 20$, Nav error = 0.1km, Time steps = [0:120:600]sec



- For the 'actual' plume shape, $\Phi_{actual,time-avg}$, we estimate a plume shape of order M_{AUV} , $\Phi_{estimated}$, using the AUV data.

Future Additions

- Plume prediction by temporal extrapolation of plume shape based on
 - Diffusion & turbulence
 - Currents
- Improve the AUV position interpolation scheme
- Link to ocean models for background environment
- Incorporate into MOOS & IvP Helm for autonomous plume tracking

Thanks!

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